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0. Introduction

In this article I present a way to model a loudspeaker driver's parameters as an electrical circuit using lumped elements. While this subject has been thoroughly explored, I will present my contribution in the form of an online calculator that will easily do the calculations for you. The article will show how the model is derived and some of its uses. Loudspeaker enclosure can also be added in the model and investigate the effects they have on the overall impedance that is present at the loudspeaker terminals.

It should be noted that this entire article refers to dynamic loudspeaker drivers only.

To try out the examples in this article yourselves you will need the following tools:

- LTSPICE: http://www.linear.com/designtools/software/
- Electric Model Calculator: http://projectryu.com/app/electric_model/
- Some driver datasheet

1. Loudspeaker's Impedance

In the beginning we should remind ourselves the nature of a driver's impedance and what are the elements that contribute to this property. This will give us some insights into the advantages and limitations the model has. In a driver we can clearly identify 3 sections corresponding to electrical, mechanical and acoustical domains and 2 energy conversions, from electrical to mechanical domain and from mechanical to acoustical domain respectively.

Figure 1. shows a representation of a driver's physical parts and an equivalent electrical circuit using lumped elements. Breaking the driver into its components allows us to easily identify what contributes to the driver's behavior. On the electrical side we have the driver's motor comprised of the voice coil and the magnetic circuit. The voice coil has a dc resistance R_e property which is given by wire material, diameter and length. Being a coil it will also have an inductance L_e which is determined in part by the voice coil's physical parameters such as size, number of wire turns, etc. The magnetic circuit has also a contribution on L_e first by creating a ferrous core and second from eddy currents that form in the steel around the voice coil.

The effect of the steel creates what is referred to as a semi-inductance and can be observed at higher frequency where the impedance rises at 3db/octave instead of 6db/octave like a normal inductor. This complicates things as the semi-inductance can not be easily modeled using lumped elements and it is not something that can be derived from fundamental properties of the driver. Some manufacturers do publish parameters that help model this but it is quite rare and many models are creating via curve matching technique which requires impedance measuring capabilities. For this reason the semi-inductance is not modeled in this online calculator.

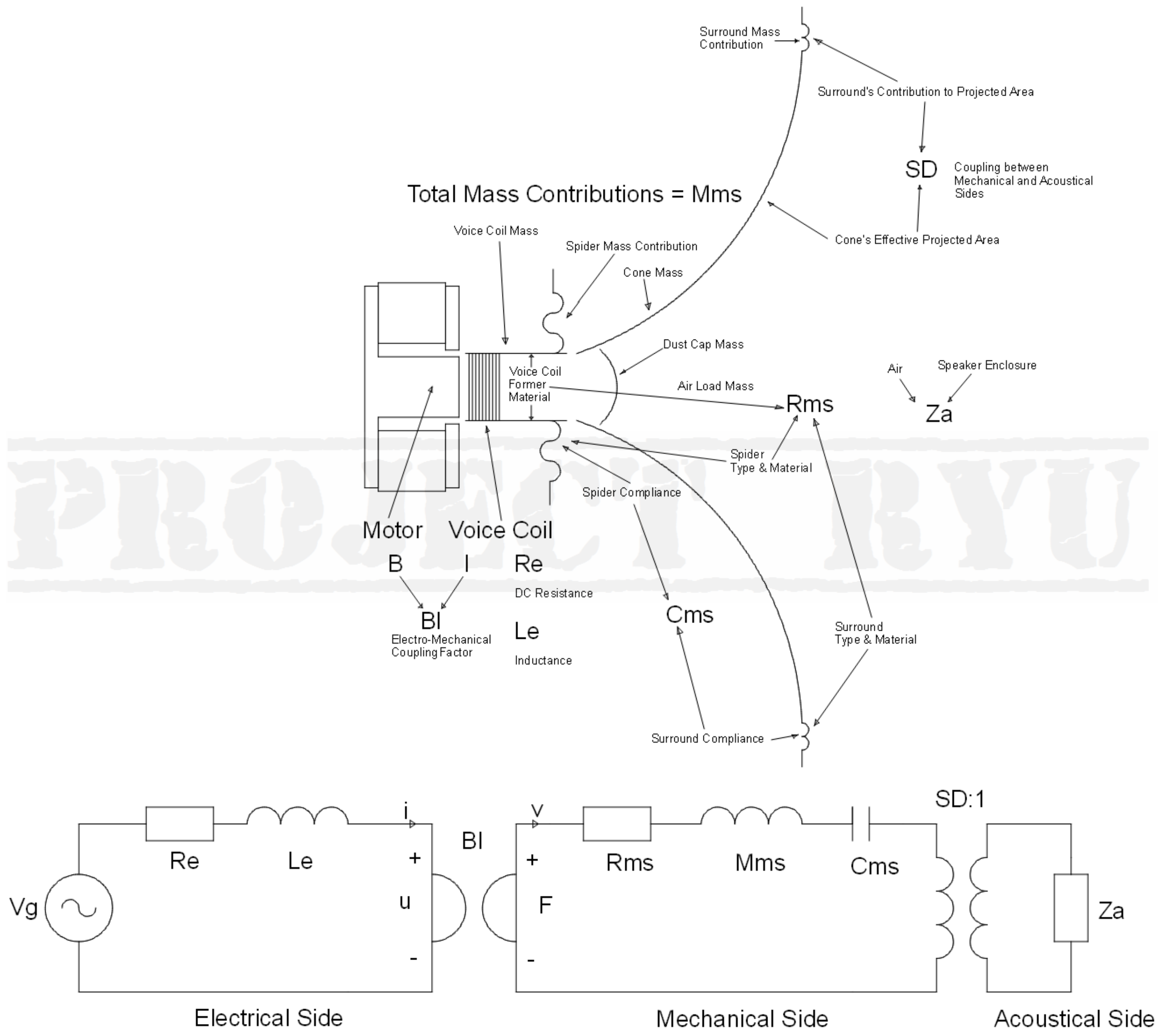


Figure 1. Loudspeaker driver model

The magnetic circuit creates a static magnetic field in the gap with a flux density B. The product between magnetic flux density B and voice coil wire length l is often called the *motor force factor* and it is basically the coupling factor between the electrical side and the mechanical side. The force exercised on the voice coil is equal to $F = Bli$ with i being the electrical current through the voice coil. We can see the coupling between the electrical side and mechanical side is therefore represented by a gyrator because we make the analogy that velocity corresponds to electrical current and force corresponds to voltage. This is referred to as impedance analogy.

If we think in terms of efficiency, than we would like to generate as much force from electrical current as possible by having a large coupling factor. Bl is the product of magnetic flux density B and voice coil wire length l but increasing the later means also increasing R_e which limits the current therefore the best way to increase the coupling factor and efficiency is to increase B.

On the mechanical side, fundamental properties such as mass and compliance are described as lumped electrical components such as resistor, inductor and capacitor. First we have the total mass of the moving elements such as the cone, dust cap and voice coil combined with the contribution to the moving mass by the spider and surround and finally the mass of the air load around the cone. Depending on the spider's and surround's geometry a fraction of their mass is added to the moving mass. This mass is represented by inductance M_{ms}.

> *Some papers treating this subject do not include the air load mass on the mechanical side and such they note the total mass as* M_{md}. *While this makes sense, it is not often used.*

Why is mass an inductance? There are two basic electric elements that can store energy, the inductance and the capacitance. To find out which one mass is, we need to apply Newton's Law of Motion stating $F = M_{ms}a$ where F is force, m is mass and a is acceleration. For instantaneous values we can say:

$$F = M_{ms}\frac{dv}{dt} \qquad (1)$$

Replacing F and v with their electrical analogues u and i we get:

$$u = L\frac{di}{dt} \qquad (2)$$

which basically describes an inductance with $L = M_{ms}$.

The compliance C_{ms} is the inverse of spider's and surround's combined stiffness and is shown as a capacitor since the relationship between force, velocity and compliance is identical to that of voltage, current and capacitance:

$$F = \frac{1}{C_{ms}}\int v dt \qquad (3)$$

The losses of the suspension system are shown as resistance R_{ms}. This is determined by the materials and geometry of the voice coil former, spider and surround. On the mechanical side we can see a resonant circuit being formed by M_{ms} and C_{ms} with $f_s = \frac{1}{2\pi\sqrt{M_{ms}C_{ms}}}$ and resistance R_{ms} acting as damping. I find it important to mention that above the resonant frequency f_s, for most of the bandwidth, the output of the driver is controlled by the mass M_{ms}.

Besides the mass contribution, the driver's cone and it's surface act as the transformer between the mechanical side and the acoustical side. The bigger the cone's surface the more mechanical energy is transformed into acoustical energy. In terms of efficiency it is desirable to have a large cone as long as the mass does not increase. Depending on its geometry part of the surround surface also contributes to the total area SD.

On the acoustical side, in figure 1, we have a generic impedance element Z_a. This corresponds to the radiation resistance for both front and back of the cone and the enclosure impedance. If no enclosure is modeled we would only keep the radiation resistance R_a and treating the cone as pulsating half sphere, we can define R_a as:

$$R_a = \frac{\rho\omega^2}{2\pi c}\frac{1}{1+(\frac{\omega r_d}{c})^2} \qquad (4)$$

$$r_d = \sqrt{\frac{SD}{2\pi}}$$

where,

ρ is air density,

c is speed of sound in air,

r_d is radius of a half sphere with same surface area as SD.

As can be seen in equation 4, R_a is non-linear and depends on frequency and thus it cannot be modeled using lumped elements. Let’s assume we have a 12″ driver with $SD = 0.056m^2$. Figure 2 plots the value of the radiation resistance on the acoustical side as R_a and after transformation to the mechanical side as R_{ma}. Values of R_{ma} are quite small and we can neglect its influence especially when R_{ms} is much higher.

The transformation from acoustical side to mechanical side is straight forward:

$$R_{ma} = SD^2 R_a \qquad (5)$$

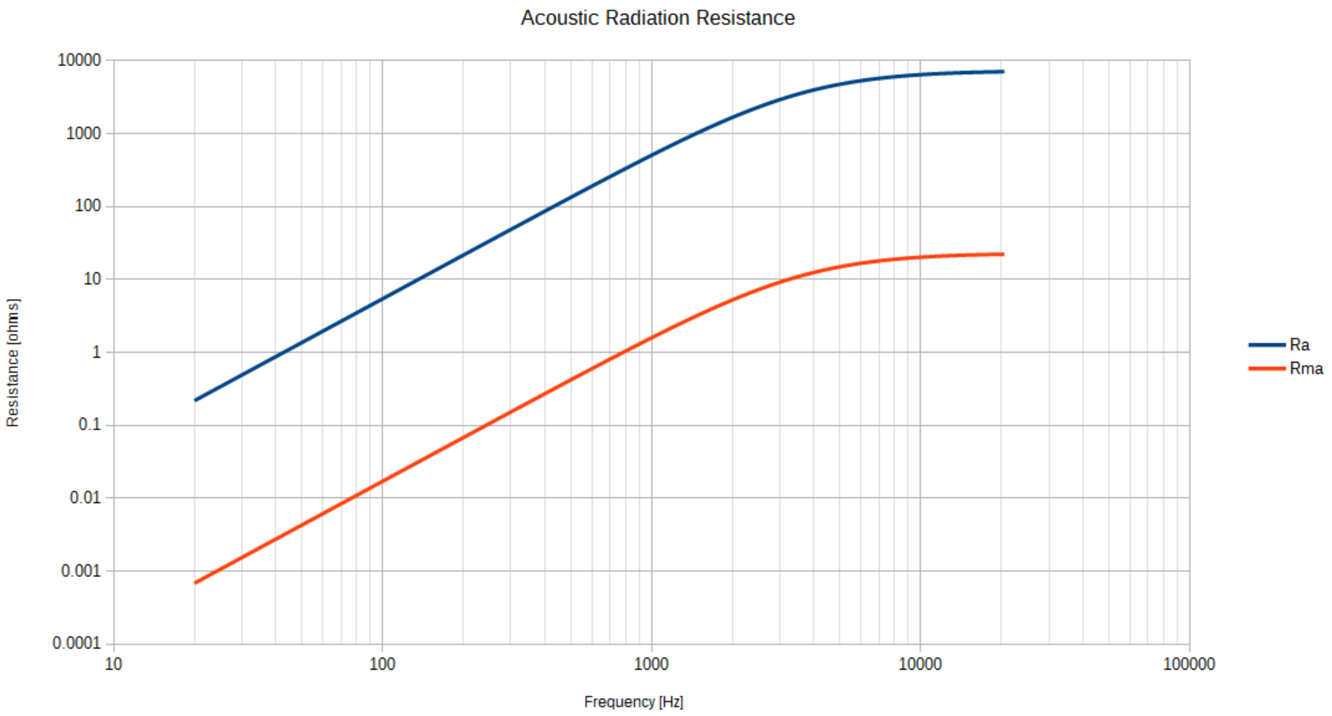


Figure 2. Acoustic Radiation Resistance for a Driver with SD = 0.056 m^2

Because of it's relatively low value and the fact that it cannot be modeled as a pure resistance, we will leave R_a out of the current model so we will only concern ourselves with the enclosure's elements on the acoustical side. If we would use M_{md} instead of Mms on the mechanical side then we should include the air mass around the front and back of the cone as inductors.

Figure 3 shows the electrical circuit equivalents for closed and vented enclosures. With a closed enclosure the air inside acts as a compliance and thus it is modeled as a capacitor C_{ab} and is defined by equation 6. If the enclosure is not air tight some of the air will leak. These losses are modeled as a resistor R_{ab} whose value depends on the parameter Q_l. With a vented enclosure we need to add the mass of air that occupies the port's volume as inductor L_{ap}. This air acts as a mass because it will move back and forth as the cone vibrates unlike the air inside the enclosure who is elastic compressing or rarefying under force and recovering when the force is removed.

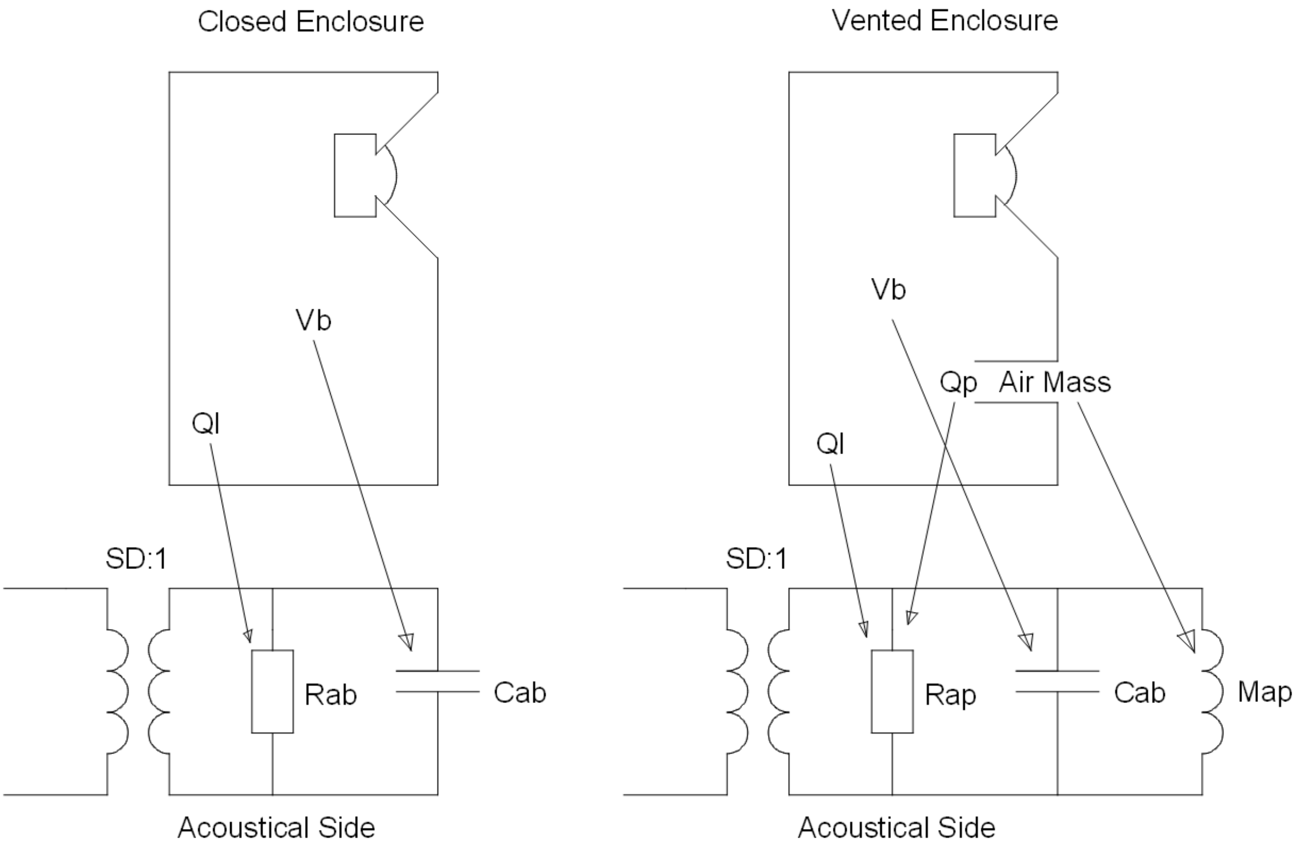


Figure 3. Loudspeaker Enclosure Model

$$C_{ab} = V_b / \rho c^2 \qquad (6)$$

$$L_{ap} = \rho l_v / S_v \qquad (7)$$

with,

V_b is enclosure volume,

ρ is density of air,

c is speed of sound,

l_v: is port lenght,

S_v: is port area

The circuit in figure 1 while offers an insight into the role each element plays, it’s not in a useful form due to the BL and SD transformations. To study the electrical impedance it would make sense to move all the elements on the electrical side and then apply circuit theory. The transformation from the acoustical side to mechanical side is defined by the following equation:

$$Z_m = SD^2 Z_a \qquad (8)$$

where,

Z_m: is the impedance on the mechanical side,

Z_a: is the impedance on the acoustical side.

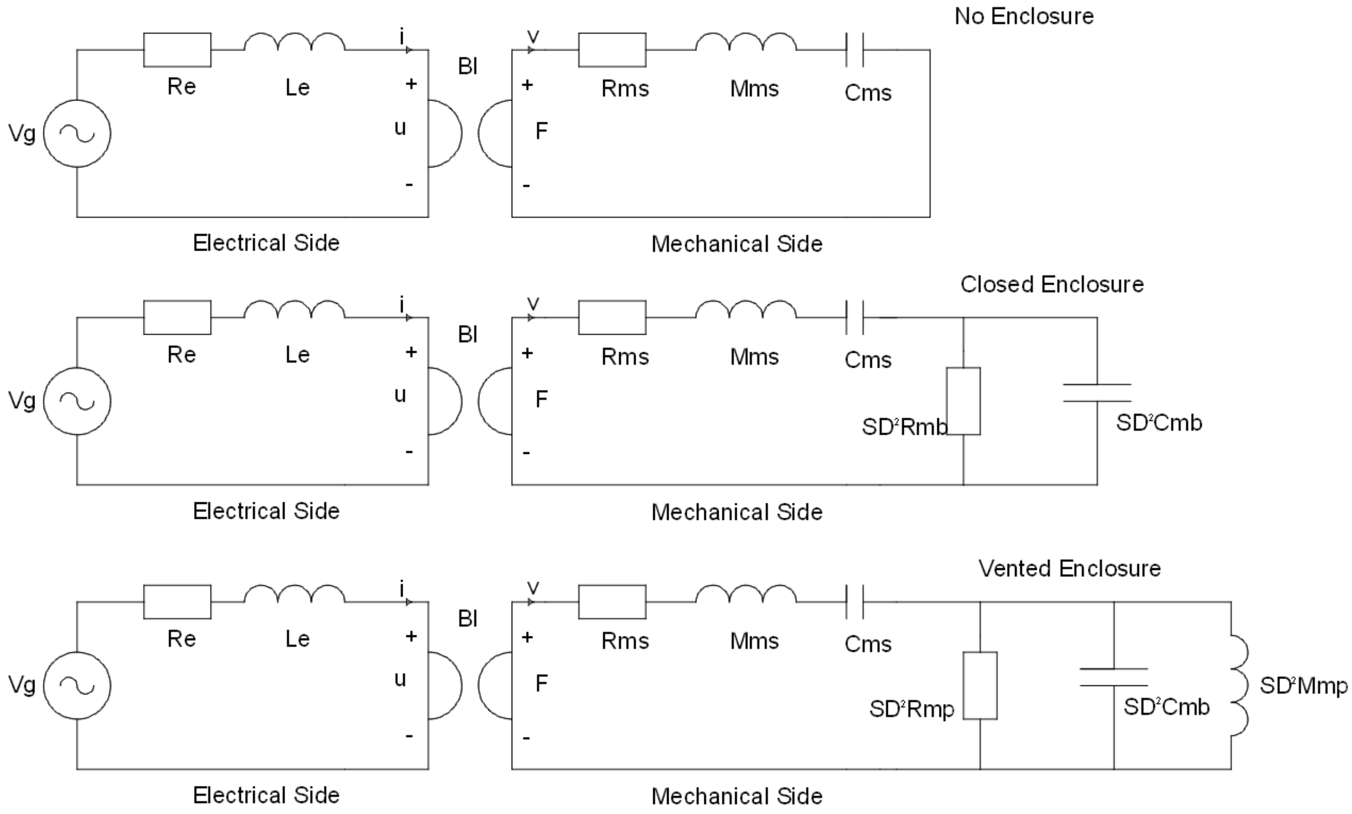


Figure 4. Converting components from acoustical side to mechanical side

The transformation from the mechanical side to electrical is a little bit more complicated due to the gyrator. It will convert series circuits to parallel circuits and vice-versa and it will convert inductors to capacitors and vice-versa. Figure 5 shows the equivalent electrical circuit with all elements transferred on the electrical side. The formulas for the conversion are listed below:

$$R_{es} = \frac{(Bl)^2}{R_{ms}} \tag{9}$$

$$C_{mes} = \frac{M_{ms}}{(Bl)^2} \tag{10}$$

$$L_{ces} = (Bl)^2 C_{ms} \tag{11}$$

$$R_{eb} = \frac{(Bl)^2}{SD^2 Rmb} \tag{12}$$

$$L_{ceb} = \frac{(Bl)^2 C_{mb}}{SD^2} \tag{13}$$

$$R_{ep} = \frac{(Bl)^2}{SD^2 Rmp} \tag{14}$$

$$C_{mep} = \frac{SD^2 M_{mp}}{(Bl)^2} \tag{15}$$

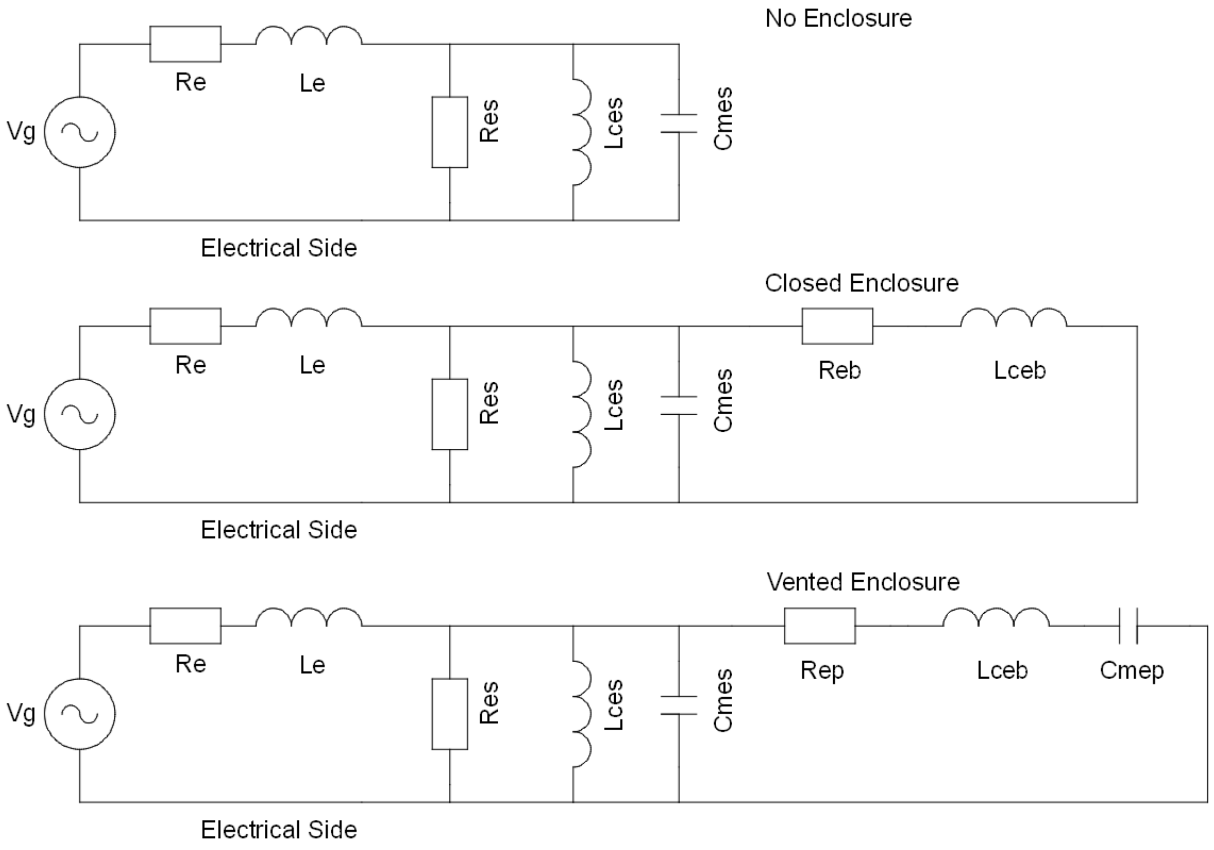


Figure 5. Converting components from mechanical side to electrical side

With no gyrators or transformers in our circuit we can easily calculate the impedance presented at the generator V_g. Using a tool such as LTSpice we can create the above circuit and define our desired simulation. To get the impedance presented at the generator we should define an ac simulation and plot $\frac{V_g}{I_g}$.

Analytically, we can brake the circuit and deal with groups of parallel or series connections one at a time. Let’s consider the vented enclosure circuit. The impedance of the series elements corresponding to the enclosure will be treated in parallel with the driver’s components and the resultant impedance will be treated in series with the voice coil components. Expressing the result in Cartesian form we get the following:

$$Z_m = R_m + jX_m \qquad (16)$$

$$R_m = \frac{1}{R_{es}} + \frac{R_{ep}}{R_{ep}^2 + (\omega L_{ceb} - \frac{1}{\omega C_{mep}})^2} \qquad (17)$$

$$X_m = \frac{1}{\omega L_{ces}} + \frac{\omega L_{ceb} - \frac{1}{\omega C_{mep}}}{R_{ep}^2 + (\omega L_{ceb} - \frac{1}{\omega C_{mep}})^2} - \omega C_{mes} \qquad (18)$$

$$Z_e = R_e + j(\omega L_e) \qquad (19)$$

$$Z_t = (R_e + R_m) + j((\omega L_e) + X_m) \qquad (20)$$

$$|Z_t| = \sqrt{(R_e + R_m)^2 + ((\omega L_e) + X_m)^2} \qquad (21)$$

$$\Phi_{Z_t} = \arctan\left(\frac{(\omega L_e) + X_m}{R_e + R_m}\right) \qquad (22)$$

where,

Z_m is the total equivalent impedance of the parallel circuit to the right of L_e,

R_m is the real part of Z_m,

jX_m is the imaginary part of Z_m,

Z_e is the total equivalent impedance of R_e and L_e,

Z_t is the total equivalent impedance presented at the generator V_g,

$|Z_t|$ is the magnitude of Z_t,

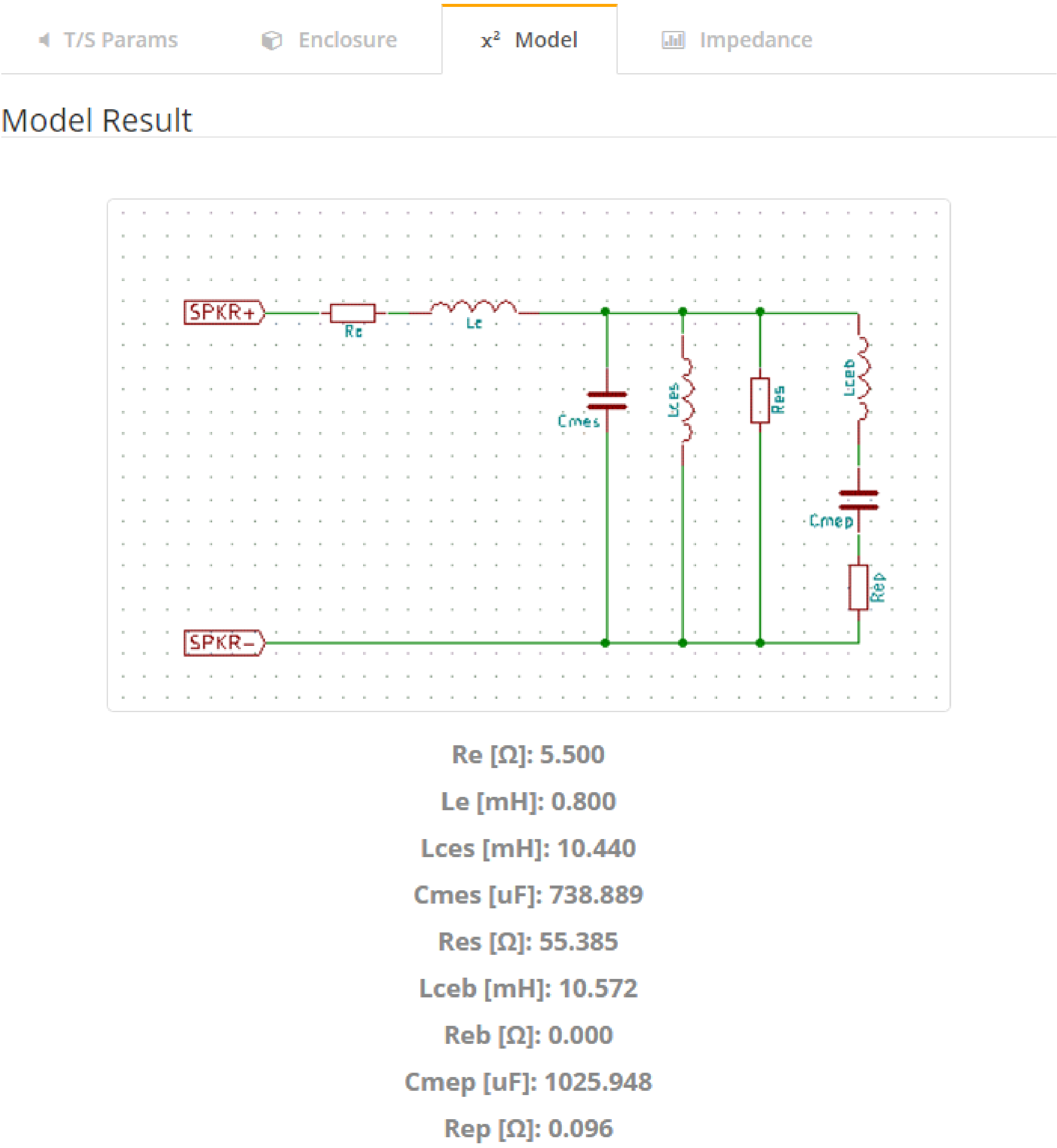
Φ_{Z_t} is the phase of Z_t.

2. Why is this useful?

For our examples let’s consider the following driver:

Resonance Frequency, Fs [Hz]:	57
DC Resistance, Re[Ω]:	5.5
Inductance, Le[mH]:	0.8
Motor Force Factor, BL[Tm]:	6
Compliance, Cms[mm/N]:	0.29
Mechanical Loses, Rms[kg/s]:	0.65
Moving Mass, Mms[g]:	26.6
Piston Surface, SD[m^2]:	0.049

Using the online calculator, let’s build the equivalent circuit model for this loudspeaker. Let say this loudspeaker will be in a vented box with a volume of 100 liters, a vent of 50 mm in length and 70 mm in diameter.



Model Result

Re [Ω]: 5.500

Le [mH]: 0.800

Lces [mH]: 10.440

Cmes [uF]: 738.889

Res [Ω]: 55.385

Lceb [mH]: 10.572

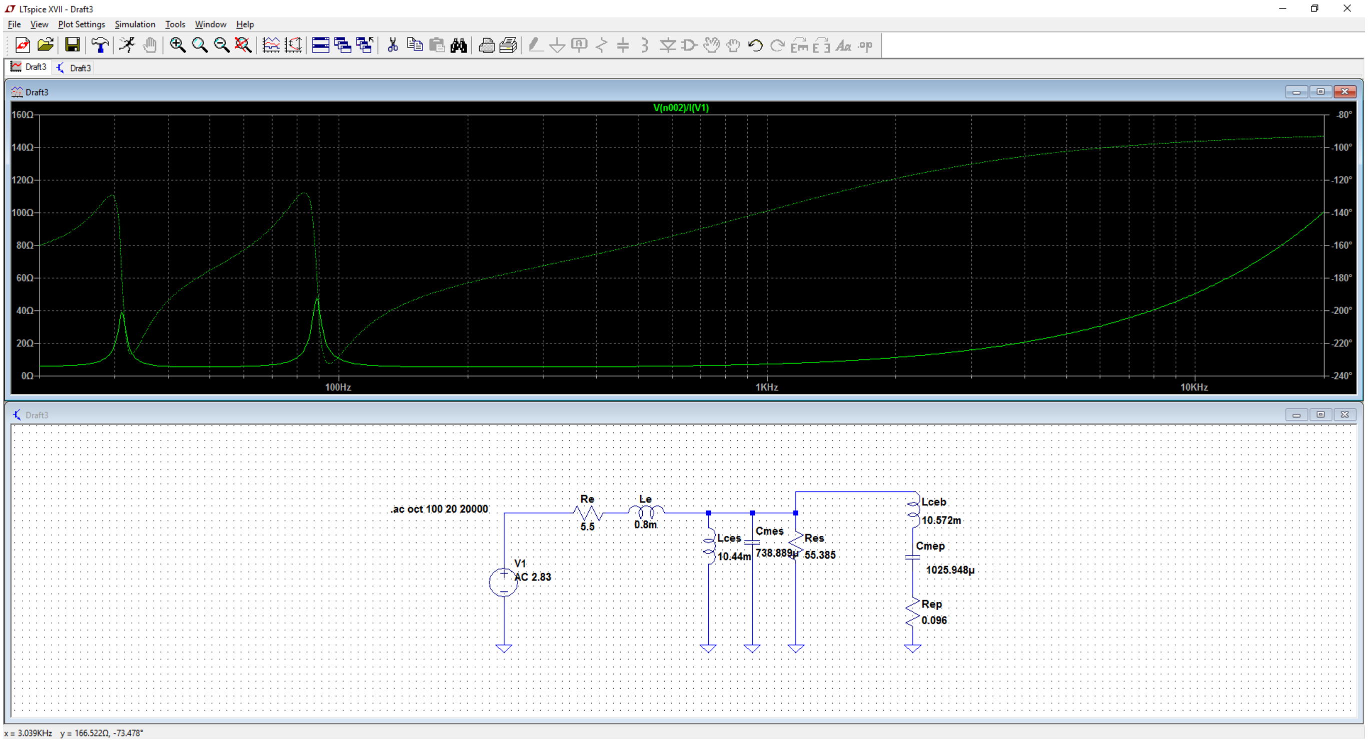
Reb [Ω]: 0.000

Cmep [uF]: 1025.948

Rep [Ω]: 0.096

Figure 6. Speaker Equivalent Electrical Circuit

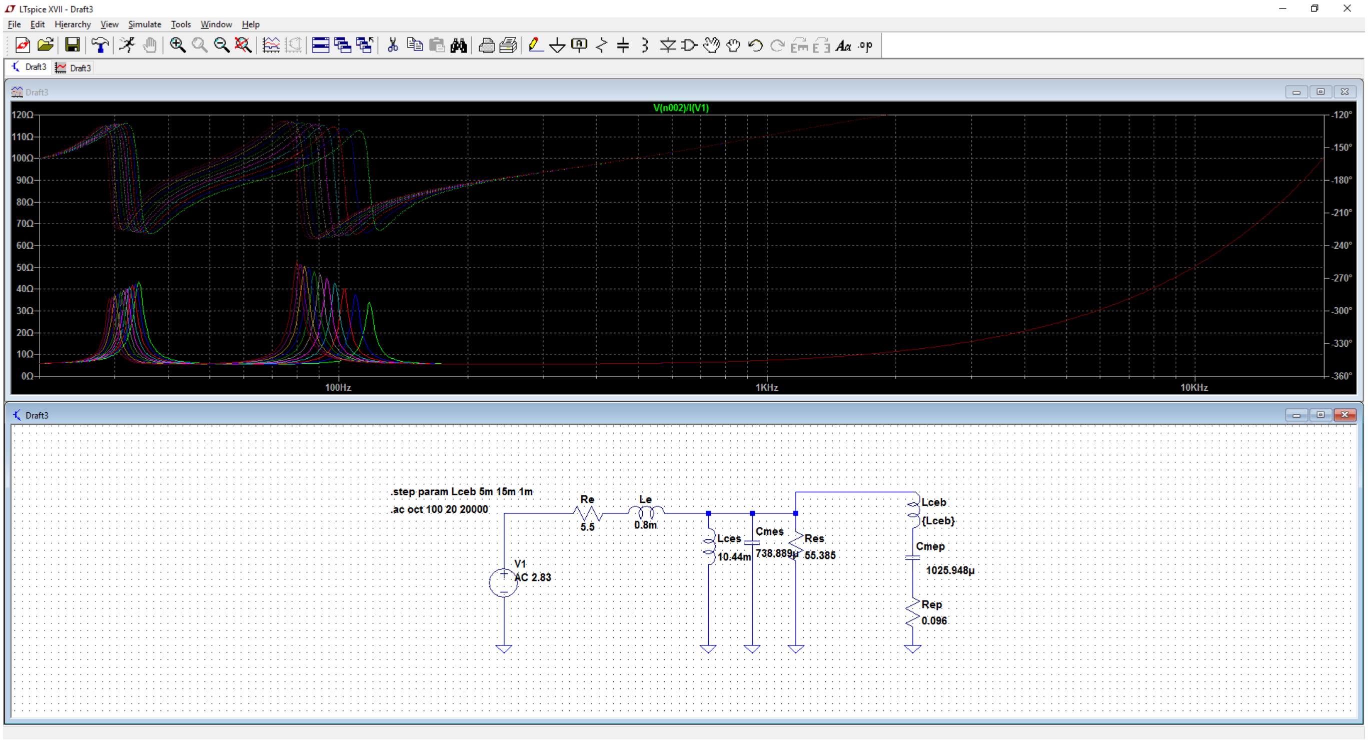
The online calculator will provide the values for all the components in the circuit. All we need to do now is to build the circuit in LTSpice and run an ac simulation. To get the impedance curve in figure 7, we will plot $\frac{V_g}{I_g}$. The resultant curve is the familiar vented loudspeaker impedance.



```
.ac oct 100 20 20000
```

Figure 7. Loudspeaker LTSpice Simulation

We can continue our analysis for example by stepping any component value from the circuit and see how the impedance is affected. In this case, let's vary L_{ceb} which corresponds to the enclosure's volume. This can be achieved by adding a $.step$ directive in LTSpice and defining the min, max and interval parameters. The result in figure 8 shows how we can easily check the enclosure volume for our desired LF alignment.

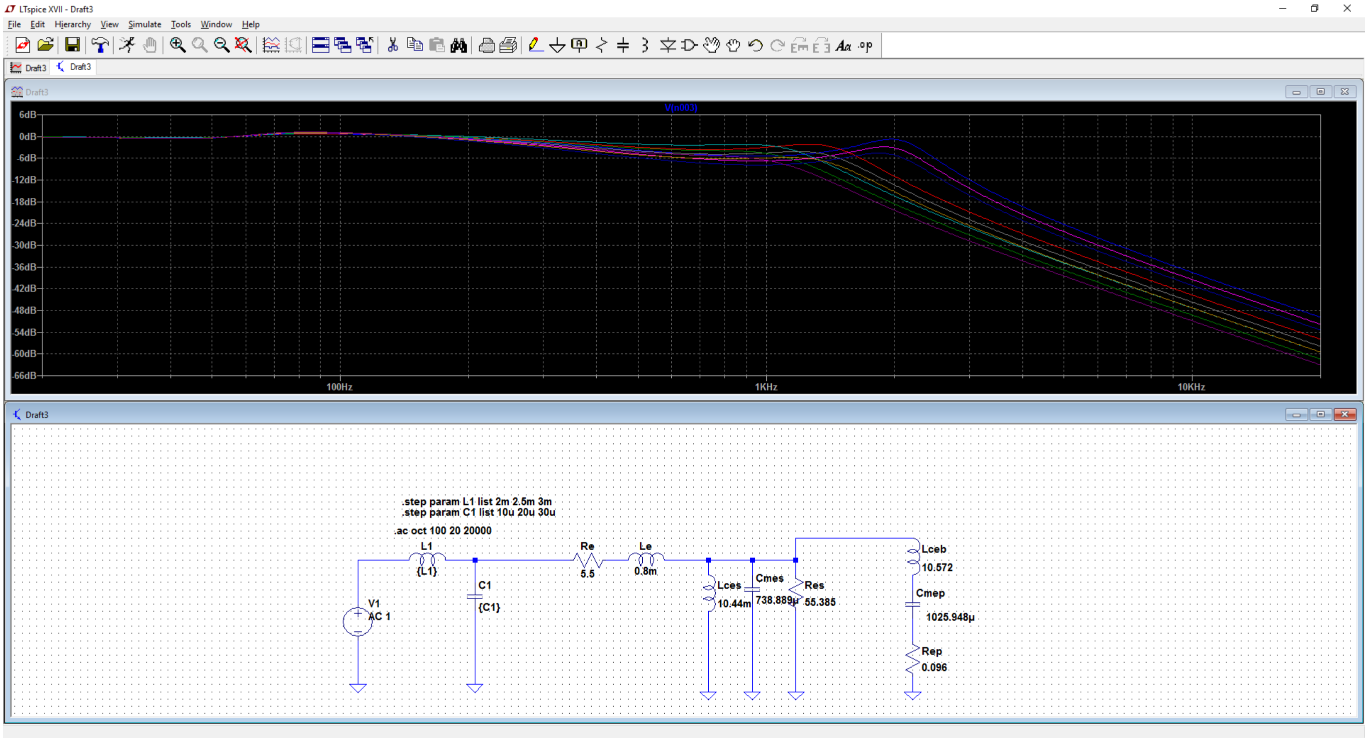


```
.step param Lceb 5m 15m 1m
.ac oct 100 20 20000
```

Figure 8. Parameter Step in Loudspeaker Simulation

Another use is to help design crossover sections. Let's look at a 2nd order low pass filter with 1kHz cutoff. We will build the circuit in LTSpice and we will simulate it's response over the audio range. As can be observed in figure 9 our crossover section is made up of inductor L_1 and capacitor C_1. We can use $.step$ again to simulate the response with different values for $L1$ and $C1$.

We can also use the equivalent electrical circuit of a loudspeaker to test amplifier output stages and check it's stability under a complex load.



```
.step param L1 list 2m 2.5m 3m
.step param C1 list 10u 20u 30u
.ac oct 100 20 20000
```

Figure 9. Low Pass Filter Simulation

3. How good is it?

The model depends on the accuracy of the T/S parameters up to the midrange. At higher frequency ωL_e term from equation 21 dominates in our model but for a real driver, as discussed in paragraph 1, the effect of Eddy currents in steel plates and/or the presence of shorting rings will combat this and L_e appears as a semi-inductance keeping the impedance lower as can be seen in figure 10. This semi-inductance is very hard to model using lumped elements and has been the subject of some debate. We will address it in future articles.

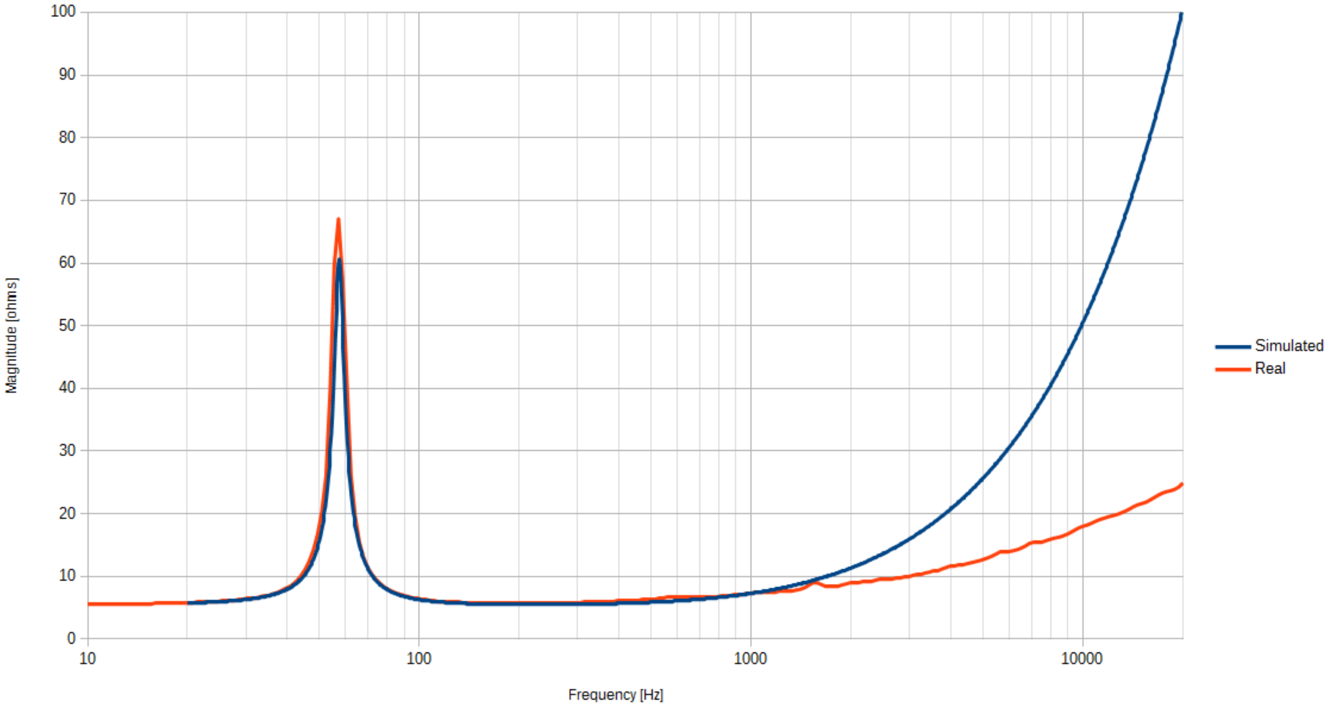


Figure 10. Simulated vs Real Loudspeaker Impedance

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